Implementing the 50% rule

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1 Introduction

In this White Paper, we will introduce an effective and efficient way of implementing the 50% rules that are defined in various sanction regimes. We will also show how to handle the case of circular ownership – an important aspect that we suspect is often missed by vendors in the compliance market. The described approach could be applied for any threshold, and its application is of course not limited to the sanction landscape.

The 50% rule is a way of defining *sanctioned by extension*. In its simplest form, it states that any entity owned directly at least 50% by sanctioned entities is itself sanctioned. The definition is applied recursively, so that children, grand-children, great-grand-children and so forth of sanctioned entities are all becoming sanctioned.

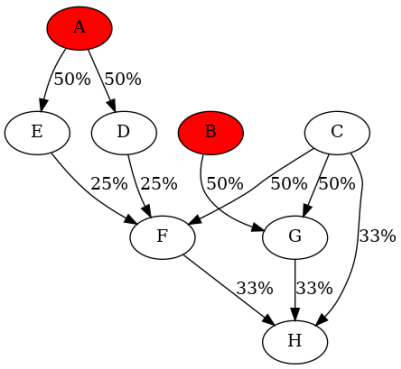

Figure 1.1: Ownership structure

Example 1.1. *In Figure 1.1, assume that* A *and* B *are sanctioned according to a sanction list.* D, E *and* G *are then all sanctioned by the 50% rule since they are directly owned at least 50% by a sanctioned company. Further,* F *is also sanctioned since it in sum is owned 50% by the sanctioned companies* D *and* E*. Finally,* H *also becomes sanctioned since it is owned 66% by the sanctioned entities* F *and* G*. Note that the non-sanctioned company* C *has an integrated ownership of 66% in* H*, but* H *still becomes sanctioned by the 50% rule.*

The OFAC 50% rule looks strictly to ownership – not control. Other sanction regimes, like the EU and the UK, refers to both ownership and to control. We will look into both aspects.

As always when analyzing ownership structures, it is important to take self ownership into account, originating from either directly self owned shares or indirect ownership through other entities.

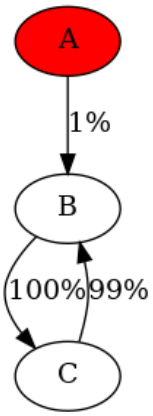

Figure 1.2: Ownership structure with Strongly Connected Component

Example 1.2. *In Figure 1.2, it is obvious that* A *is the sole ultimate owner of both* B *and* C*. If* A *is sanctioned, so should* B *and* C *be. However, applying the 50% rule naively will not sanction the other entities – we need to gross up the ownership link from* A *to* B *based on* B*'s self ownership.*

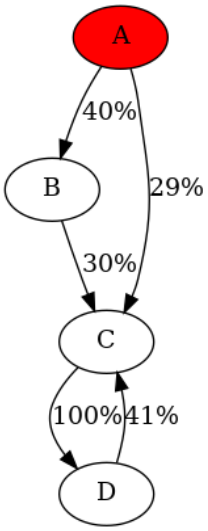

Figure 1.3: Another ownership structure with Strongly Connected Component

Example 1.3. *In Figure 1.3, it is not quite as obvious whether* C *and thus also* D *is sanctioned by extension or not.* B *is not sanctioned by extension.* A*'s integrated ownerhip in* C *is 73.15%, but a lot of this ownership is coming through* B*, and this part shouldn't be included when we consider* A*'s direct ownership only. The direct ownership contributes with* $\frac{29}{100-41} = 49.15\%$*, so that* C *and* D *avoid sanctioning.*

2 Implementation

In this White Paper, we will skip proofs and detailed explanations as to why the proposed algorithms work as anticipated. However, we believe that the algorithm is simple enough that the interested reader should have no problem understanding the underlying mechanism.

2.1 Ownership only

Let O be the standard $n \times n$ ownership matrix where o_{ij} is the ownership entity i has in entity j.

From this matrix, we will create an *adjusted ownership matrix*, A, where we have adjusted every element o_{ij} according to any self ownership in entity j. Let E be a matrix containing all integrated ownership values. We then build A using the following formula:

$$a_{ij} = \begin{cases} \frac{o_{ij}}{(1-(e_{jj}-e_{ji}\times o_{ij}))}, & \text{if } i \neq j \\ 1, & \text{otherwise} \end{cases} \quad (1)$$

The e_{jj} term in Eq(1) corrects for any self-ownership of firm j. The other term in the denominator corrects for any circular ownership paths of firm j via firm i.

Further, we create an n-dimensional vector s^1 like this:

$$s_i^1 = \begin{cases} 1, & \text{if entity } i \text{ is sanctioned} \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

Finally, we define a *matrix* $\odot$ *vector* $\rightarrow$ *vector* operator, where the i-th element of the result vector – r_i – when doing $B \odot x$ is obtained using the following formula:

$$r_i = \begin{cases} 1, & \text{if } b_{i*} \cdot x \geq 0.5 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where b_{i*} is the i-th row of B and $\cdot$ is the standard dot product. We put all together and arrive at the final equation:

$$s^{n+1} = A^T \odot s^n, \text{repeat until } s^{n+1} = s^n \quad (4)$$

where A^T is the transpose of the adjusted ownership matrix A. What we obtain with this is that, firstly, the new sanction vector s^{n+1} obtained in each iteration, will always have 1 for entries that had 1 in the previous vector s^n since we assigned 1's to the diagonal of A. For entries other than those being marked as sanctioned in s^n, the dot product will yield the sum of direct ownership for the entites which have been found to be sanctioned. If this sum is greater than the threshold, we mark the entry as sanctioned by assigning 1 in the right position in the vector s^{n+1}.

When the process terminates ($s^{n+1} = s^n$), all sanctioned entities will have been marked by a 1 in s^n.

2.2 Control only

We can apply the exact same algorithm for control as for ownership, we just have to replace the adjusted ownership matrix A with a control matrix C that is created from O like this:

1. If voting rights are not equal to ownership, the corresponding columns should contain voting right shares rather than ownership shares.
2. Assign 1 to every element on the diagonal just like for A.
3. If *Control by other means* should be taken into account, a_{ij} should be assigned 1 whenever entity i controls entity j in any way, e.g. "has the right to appoint more than half of the board members" if that is one of the criteria. Note that there is no restriction on the sum of values for a column in C, the algorithm will work even if several entities are said to control another entity.
4. T-rank would advocate that Voting Power should be taken into account. If entity i has sufficient Voting Power in entity j to be considered as De Facto in control, a 1 should be assigned.
5. No adjustments for integrated ownership.

2.3 Combining ownership and control

How we combine the two depends on the *50% rule* definition.

If the definition states that an entity falling under the 50% rule, either by looking at ownership or by looking at control, is sanctioned by extension, the calculation is just a matter of doing the respective calculations seperately and finally creating the result vector as the logical union of the two result vectors obtained.

If, on the other hand, the definition opens up for alternations between control and ownership on different levels of ownership chains (e.g A is sanctioned; A owns 50% of B; B has 50% of the voting rights in C), we need to allow for both ownership and control in each iteration, e.g.:

$$s^{n+1} = C^T \odot (A^T \odot s^n) \quad (5)$$